

Excited $(70, L^+)$ baryon resonances in the relativistic quark model

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Abstract

The masses of positive parity $(70, 0^+)$ and $(70, 2^+)$ nonstrange and strange baryons are calculated in the relativistic quark model. The relativistic three-quark equations of the $(70, L^+)$ multiplets are found in the framework of the dispersion relation technique. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. The calculated mass values of the $(70, L^+)$ multiplets are in good agreement with the experimental ones.

PACS: 11.55.Fv, 12.39.Ki, 12.40.Yx, 14.20.-c.

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1. Introduction.

Hadron spectroscopy has always played an important role in the revealing mechanisms underlying the dynamic of strong interactions.

At low energies, typical for baryon spectroscopy, QCD does not admit a perturbative expansion in the strong coupling constant. In 1974 't Hooft [1] suggested a perturbative expansion of QCD in terms of the parameter $1/N_c$ where N_c is the number of colors. This suggestion together with the power counting rules of Witten [2] has lead to the $1/N_c$ expansion method which allows to systematically analyse baryon properties. The success of the method stems from the discovery that the ground state baryons have an exact contracted $SU(2N_f)$ symmetry when $N_c \rightarrow \infty$ [3, 4], N_f being the number of flavors. For $N_c \rightarrow \infty$ the baryon masses are degenerated. For large N_c the mass splitting starts at order $1/N_c$. Operator reduction rules simplify the $1/N_c$ expansion [5, 6].

A considerable amount of work has been devoted to the ground state baryons, described by the symmetric representation 56 of $SU(6)$ [7-11]. The excited baryons belonging to $(56, L)$ multiplets can be studied by analogy with the ground state. In this case both the orbital and the spin-flavor parts of the wave functions are symmetric. Explicit forms for such wave functions were given, for example, in Ref [12]. Together with color part, they generate antisymmetric wave functions. Naturally, it turned out that the splitting starts at order $1/N_c$ as for the ground state.

The states belonging to $(70, L)$ multiplets are apparently more difficult. In this case the general practice was to split the baryon into excited quark and a symmetric core, the latter being either in the ground state for the $N = 1$ or in an excited state for the $N \geq 2$ bands. Recently Matagne and Stancu have suggested the new approach [13] for the excited $(70, 1^-)$ multiplet. They solved the problem by removing the splitting of generators and using orbital-flavor-spin wave functions. The excited baryons are considered as bound states. The basic conclusion is that the first order correction to the baryon masses is order $1/N_c$ instead of order N_c^0 as previously found. The conceptual difference between the ground state and the excited states is therefore removed.

The constituent quark model suggests that $(70, L^+)$ baryons are composed of the system of N_c quarks, which is divided into an excited quark and a core, which can be excited or not [14]. The standart procedure, used in Ref. [15], for calculating the mass spectrum is to reduce the wave function to that of a product of a symmetric orbital and symmetric flavor-spin wave function for the core of $N_c - 1$ quarks times the wave function of the excited quark. This implies that the total orbital-flavor-spin wave function is truncated to a single term, described by the product of two Young tableaux, each with the excited quark in the second row. Many others terms, related to Young tableaux with the excited quark in the first row, are neglected [16]. At the same time each $SU(6)$ and $O(3)$ generators is splitted into two terms, are acting on the core and the other on the excited quark. It is not possible to treat the $(70, L^+)$ multiplet composed of strange baryons without simplifying the baryon wave function and splitting of $SU(6)$ generators.

In the series of papers [17-21] a practical treatment of relativistic three-hadron systems have been developed. The physics of three-hadron system is usefully described in term of the pairwise interactions among

the three particles. The theory is based on the two principles of unitarity and analyticity, as applied to the two-body subenergy channels. The linear integral equations in a single variable are obtained for the isobar amplitudes. Instead of the quadrature methods of obtaining solution the set of suitable functions is identified and used as basis set for the expansion of the desired solutions. By this means the couple integral equations are solved in terms of simple algebra.

In our papers [22, 23] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectrum of S -wave baryons including u , d , s -quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such an approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In our paper [24] the construction of the orbital-flavor-spin wave functions for the $(70, 1^-)$ multiplet are given. We deal with a three-quark system having one unit of orbital excitation. The orbital part of wave function must have a mixed symmetry. The spin-flavor part of wave function must have the same symmetry in order to obtain a totally symmetric state in the orbital-flavor-spin space. The integral equations using the orbital-flavor-spin wave functions was constructed. It allows to calculate the mass spectra for all baryons of $(70, 1^-)$ multiplet. We take into account the u , d , s -quarks. We have represented the 30 nonstrange and strange resonances belonging to the $(70, 1^-)$ multiplet. The 15 resonances are in good agreement with experimental data [25]. We have predicted 15 masses of baryons. In our model the four parameter are used: gluon coupling constants g_+ and g_- for the various parity, cutoff energy parameters λ , λ_s for the nonstrange and strange diquarks.

The paper is organized as follows. After this introduction, we discuss the construction of the orbital-flavor-spin wave functions for the $(70, 0^+)$ and $(70, 2^+)$ multiplets.

In Sect. 3 the relativistic three-quark equations are obtained in the form of the dispersion relation over the two-body subenergy.

In Sect. 4 the systems of equations for the reduced amplitudes are derived.

Section 5 is devoted to the calculation results for the mass spectrum of the $(70, 0^+)$ and $(70, 2^+)$ multiplets (Tables I-XII).

In Conclusion, the status of the considered model is discussed.

In Appendix I the wave functions of $(70, 0^+)$ and $(70, 2^+)$ baryon resonances are given.

In Appendix II the reduced equations for the $(70, 0^+)$ and $(70, 2^+)$ multiplets are obtained.

2. The wave function of $(70, 0^+)$ and $(70, 2^+)$ excited states.

The multiplet $(70, 2^+)$ consists of the excited baryon resonances with the orbital moment $L = 2$ and positive-parity. According to the nonrelativistic approach [15], the $(70, 2^+)$ multiplet states includes the two quarks on the $1s$ -levels and one quark on the $1d$ -level (002) or the two quarks on the $1p$ -levels and one quark on the $1s$ -level (110). Then the baryons of multiplet $(70, 2^+)$ consists of the superposition of the 002 and 110 states. The transition of these states with the orbital moment $L_z = 2$ are considered.

The multiplet $(70, 2^+)$ of $SU(6)$ includes the decuplet $(10, 2)$ with the spin $S = \frac{1}{2}$, octet $(8, 2)$ with the spin $S = \frac{1}{2}$, octet $(8, 4)$ with the spin $S = \frac{3}{2}$ and singlet $(1, 2)$ with the spin $S = \frac{1}{2}$. Taking into account the orbital and spin moment $\vec{J} = \vec{L} + \vec{S}$, we obtain the angular moment for the $S = \frac{1}{2}$ $J = \frac{3}{2}, \frac{5}{2}$ and for the $S = \frac{3}{2}$ $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$. We can represent the total multiplet $(70, 2^+)$:

$$\begin{array}{llll}
(10, 2) : & \frac{3}{2}^+, & \frac{5}{2}^+ & P_{33}, \quad F_{35} \\
(8, 2) : & \frac{3}{2}^+, & \frac{5}{2}^+ & P_{13}, \quad F_{15} \\
(8, 4) : & \frac{1}{2}^+, & \frac{3}{2}^+, & \frac{5}{2}^+, \quad \frac{7}{2}^+ \quad P_{11}, \quad P_{13}, \quad F_{15}, \quad F_{17} \\
(1, 2) : & \frac{3}{2}^+, & \frac{5}{2}^+ & P_{03}, \quad F_{05}
\end{array}$$

The $(70, 2^+)$ multiplet includes the 34 baryons with different masses.

The $(70, 0^+)$ multiplet includes the excited baryon resonances with the orbital moment $L = 0$ and the positive-parity. The states of this multiplet consist of the two quarks on the $1s$ -levels and one radial excited quark on the level $2s$, or the two quarks on the $1p$ -levels with the projection of orbital moment $L_z = 0$ and

one quark on the 1s-level. We consider the spin $S = \frac{1}{2}$ and $J = \frac{1}{2}$, and $S = \frac{3}{2}$, $J = \frac{3}{2}$. We can represent the total multiplet $(70, 0^+)$:

$$\begin{array}{lll} (10, 2) : & \frac{1}{2}^+ & P_{31} \\ (8, 2) : & \frac{1}{2}^+ & P_{11} \\ (8, 4) : & \frac{3}{2}^+ & P_{13} \\ (1, 2) : & \frac{1}{2}^+ & P_{01} \end{array}$$

The $(70, 0^+)$ multiplet includes the 13 baryons with different masses.

The three-quark wave function of the excited baryon possesses the symmetry $SU(6) \times O(3) \times SU(3)_c$, where the $SU(3)_c$ group determines the color symmetry, therefore the total wave function is antisymmetric. The part of wave function $SU(6) \times O(3)$ must be total symmetric.

The $O(3)$ wave functions with the mixed symmetry allow to construct two states with the mixed symmetry and the positive-parity. Then we use these states and two mixed multiplets 70 and $70'$ of group $SU(6)$. We can construct the total symmetric state of multiplet $(70, 2^+)$.

$O(3)$ wave functions with the mixed symmetry are there:

$$\varphi_{MA}^{O(3)} = \frac{1}{2} (020 - 200 - 101 + 011), \quad \varphi_{MS}^{O(3)} = \frac{1}{\sqrt{12}} (020 + 200 - 2 \cdot 002 - 101 - 011 + 2 \cdot 110), \quad (1)$$

here 0, 1, 2 are the values of the projections of quark orbital momentum. MA and MS correspond to the mixed antisymmetric and symmetric part of wave function. $SU(6)$ wave functions are chosen for the each states. For the sake of simplicity we have derived the wave functions for the $(10, 2)$ decuplet Σ^+ .

By analogy of the paper [24]:

$$\varphi_{MA}^{SU(6)} = \varphi_S^{SU(3)} \varphi_{MA}^{SU(2)}, \quad \varphi_{MS}^{SU(6)} = \varphi_S^{SU(3)} \varphi_{MS}^{SU(2)}, \quad (2)$$

$$\varphi_{MA}^{SU(2)} = \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \varphi_{MS}^{SU(2)} = \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow), \quad (3)$$

$$\varphi_S^{SU(3)} = \frac{1}{\sqrt{3}} (usu + suu + uus). \quad (4)$$

\uparrow and \downarrow determine the spin directions. 2, 1 and 0 correspond to the excited or nonexcited quarks.

We use the functions (2) – (4) and construct the $SU(3)$ functions for each particle.

The total symmetric $SU(6) \times O(3)$ wave functions are similar to:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right) \equiv \frac{1}{\sqrt{2}} \varphi_S^{SU(3)} \left(\varphi_{MA}^{SU(2)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(2)} \varphi_{MS}^{O(3)} \right). \quad (5)$$

For instance, we have obtained the wave function for the Σ^+ of $(10, 2)$ multiplet as:

$$\begin{aligned} \varphi_{\Sigma^+(10,2)} = \frac{\sqrt{3}}{18} & (2\{u^2 \downarrow u \uparrow s \uparrow\} + \{s^2 \downarrow u \uparrow u \uparrow\} - \{u^2 \uparrow u \downarrow s \uparrow\} - \{u^2 \uparrow u \uparrow s \downarrow\} - \{s^2 \uparrow u \uparrow u \downarrow\} - \\ & - 2\{u \downarrow u^1 \uparrow s^1 \uparrow\} - \{s \downarrow u^1 \uparrow u^1 \uparrow\} + \{u \uparrow u^1 \downarrow s^1 \uparrow\} + \{u \uparrow u^1 \uparrow s^1 \downarrow\} + \{s \uparrow u^1 \uparrow u^1 \downarrow\}). \end{aligned} \quad (6)$$

Here the parentheses determine the symmetrical function:

$$\{abc\} \equiv abc + acb + bac + cab + bca + cba. \quad (7)$$

The wave functions of Σ^0 - Σ^- -hyperons can be constructed by similar way.

For the Δ baryon of $(10, 2)$ multiplet the wave function can be obtained if we replace by $u \leftrightarrow s$ quarks.

$$\varphi_{\Delta^{++}(10,2)} = \frac{1}{6} (\{u^2 \downarrow u \uparrow u \uparrow\} - \{u^2 \uparrow u \uparrow u \downarrow\} - \{u \downarrow u^1 \uparrow u^1 \uparrow\} + \{u \uparrow u^1 \uparrow u^1 \downarrow\}). \quad (8)$$

For the $\Xi^{0,-}$ of the $(10, 2)$ multiplet the wave function is similar to the $\Sigma^{+,-}$ state with the replacement by $u \leftrightarrow s$ or $d \leftrightarrow s$. The wave function for the Ω^- of the $(10, 2)$ decuplet is determined as the Δ^{++} state with the replacement by $u \rightarrow s$ quarks.

3. The three-quark integral equations for the $(70, 0^+)$ and $(70, 2^+)$ multiplets.

By consideration of the construction of $(70, 0^+)$ and $(70, 2^+)$ multiplets integral equations we need to using the projectors for the different diquark states. The projectors to the symmetric and antisymmetric states can be obtained as:

$$\frac{1}{2}(q_1 q_2 + q_2 q_1), \quad \frac{1}{2}(q_1 q_2 - q_2 q_1). \quad (9)$$

The spin projectors are following:

$$\frac{1}{2}(\uparrow\downarrow + \downarrow\uparrow), \quad \frac{1}{2}(\uparrow\downarrow - \downarrow\uparrow). \quad (10)$$

The orbital moment excitation projectors take into account the transition of diquarks $20 \leftrightarrow 11$ with the value of orbital moment projection $L_z = 2$.

$L_z = 2$:

$$200 : A^{s0+}, A^{d2+}, \quad 011 : A^{p2+}, A^{p1-}. \quad (11)$$

$L_z = 1$:

$$1^*00 : A^{s0+}, A^{d1+}, \quad 00^*1 : A^{p1+}, \frac{1}{2}(A^{p0-} + A^{p1-}). \quad (12)$$

$L_z = 0$:

$$0^*00 : A^{s0+}, A^{d0+}, \quad 00^*0^*, 01(-1)^* : A^{p0+}, \frac{1}{4}(2 \cdot A^{p0-} + A^{p1-} + A^{p(-1)-}). \quad (13)$$

$L_z = -1$:

$$(-1)^*00 : A^{s0+}, A^{d(-1)+}, \quad 00^*(-1)^* : A^{p(-1)+}, \frac{1}{2}(A^{p0-} + A^{p(-1)-}). \quad (14)$$

The upper index determines the diquark states. The excited quark is determined by (*).

11 :

$$\frac{A^{sym}}{4}(2 \cdot 11 + 20 + 02), \quad (15)$$

10 :

$$\frac{A^{sym}}{2}(10 + 01) + \frac{A^{asym}}{2}(10 - 01), \quad (16)$$

00 :

$$A^{sym} \cdot 00, \quad (17)$$

20 :

$$\frac{A^{sym}}{4}(20 + 02 + 2 \cdot 11) + \frac{A^{asym}}{2}(20 - 02), \quad (18)$$

here

$$A^{sym} = \frac{A^{s0+} + A^{p2+}}{2}, \quad A^{asym} = \frac{A^{d2+} + A^{p1-}}{2}. \quad (19)$$

The product of three projectors $SU(3)_f \times SU(2) \times O(3)$ must be symmetrical.

For example, the projector to the diquark $u^2 \uparrow s \downarrow$ is following:

$$\begin{aligned} & \frac{A_1^{sym s}}{16}(us + su)(\uparrow\downarrow + \downarrow\uparrow)(20 + 02 + 2 \cdot 11) + \frac{A_0^{sym s}}{16}(us - su)(\uparrow\downarrow - \downarrow\uparrow)(20 + 02 + 2 \cdot 11) \\ & + \frac{A_1^{asym s}}{8}(us - su)(\uparrow\downarrow + \downarrow\uparrow)(20 - 02) + \frac{A_0^{asym s}}{8}(us + su)(\uparrow\downarrow - \downarrow\uparrow)(20 - 02). \end{aligned} \quad (20)$$

here the lower index of amplitude corresponds to the diquark spin (1 or 0), and the upper index s points out the strangeness of diquark.

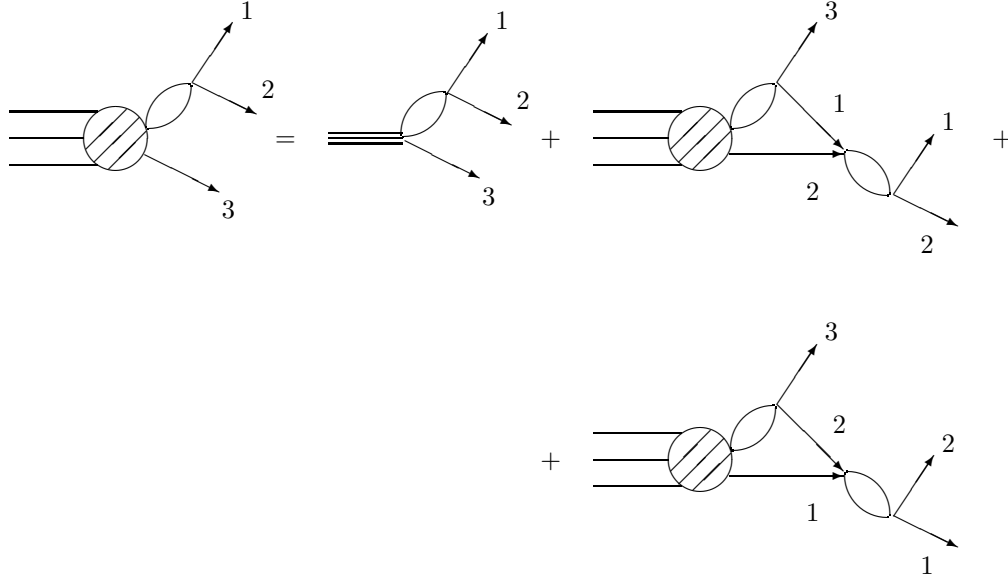


Fig.2. Graphic representation of the equations for the amplitude $A_1(s, s_{ik})$.

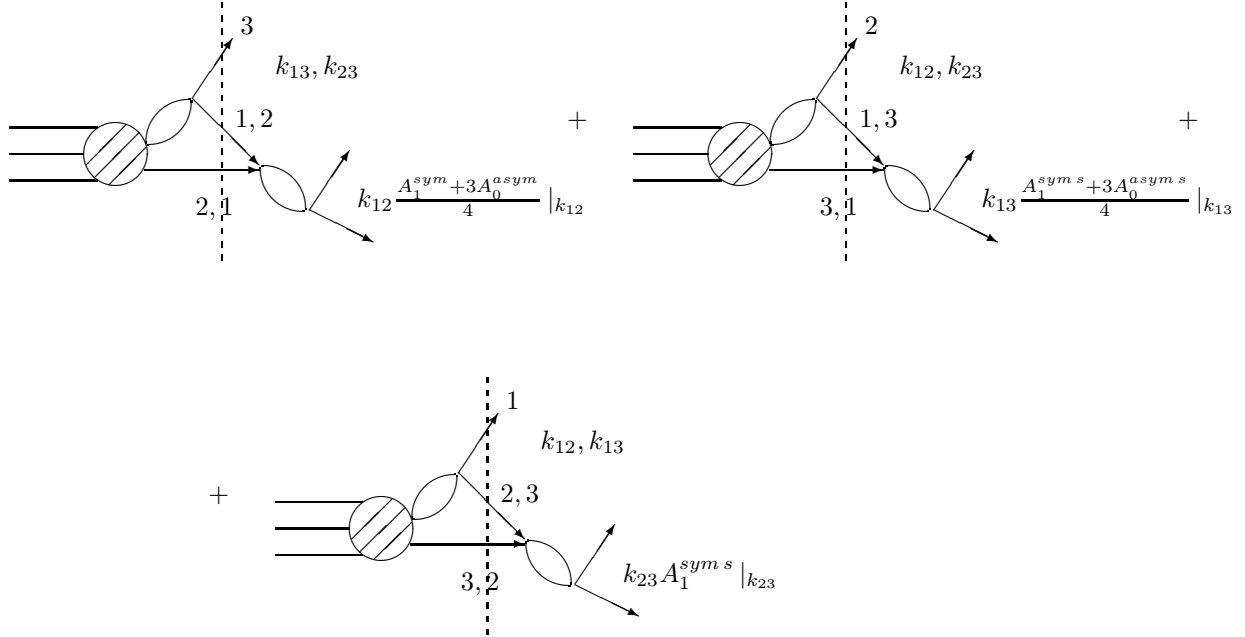


Fig.3. The contribution of the diagrams with the rescattering.

We use the diquark projectors. Then we consider the particle $\Sigma \frac{5}{2}^+$ of the $(10, 2) (70, 2^+)$ multiplet again. This wave function contains the contribution to $u^2 \downarrow u \uparrow s \uparrow$, which includes three diquarks: $u^2 \downarrow u \uparrow$, $u^2 \downarrow s \uparrow$ and $u \uparrow s \uparrow$. The diquark projectors allow us to obtain the equations (25) – (27) (with the definition $\rho_J(s_{ij}) \equiv k_{ij}$).

$$\begin{aligned}
& k_{12} \left(\frac{A_1^{s0+} + A_1^{p2+} + 2 \cdot A_0^{d2+} + 2 \cdot A_0^{p1-}}{16} (u^2 \downarrow u \uparrow s \uparrow + u \uparrow u^2 \downarrow s \uparrow) + \right. \\
& + \frac{A_1^{s0+} + A_1^{p2+} - 2 \cdot A_0^{d2+} - 2 \cdot A_0^{p1-}}{16} (u^2 \uparrow u \downarrow s \uparrow + u \downarrow u^2 \uparrow s \uparrow) \\
& \left. + \frac{A_1^{s0+} + A_1^{p2+}}{8} (u^1 \downarrow u^1 \uparrow s \uparrow + u^1 \uparrow u^1 \downarrow s \uparrow) \right), \tag{25}
\end{aligned}$$

$$\begin{aligned}
& k_{13} \left(\frac{A_1^{s0s+} + A_1^{p2s+} + 2 \cdot A_0^{d2+} + 2 \cdot A_0^{p1-} + A_0^{s0s+} + A_0^{p2s+} + 2 \cdot A_1^{d2+} + 2 \cdot A_1^{p1-}}{32} (u^2 \downarrow u \uparrow s \uparrow + s \uparrow u \uparrow u^2 \downarrow) + \right. \\
& + \frac{A_1^{s0s+} + A_1^{p2s+} - 2 \cdot A_0^{d2+} - 2 \cdot A_0^{p1-} - A_0^{s0s+} - A_0^{p2s+} + 2 \cdot A_1^{d2+} + 2 \cdot A_1^{p1-}}{32} (u^2 \uparrow u \uparrow s \downarrow + s \downarrow u \uparrow u^2 \uparrow) + \\
& + \frac{A_1^{s0s+} + A_1^{p2s+} + 2 \cdot A_0^{d2+} + 2 \cdot A_0^{p1-} - A_0^{s0s+} - A_0^{p2s+} - 2 \cdot A_1^{d2+} - 2 \cdot A_1^{p1-}}{32} (s^2 \downarrow u \uparrow u \uparrow + u \uparrow u \uparrow s^2 \downarrow) + \\
& + \frac{A_1^{s0s+} + A_1^{p2s+} - 2 \cdot A_0^{d2+} - 2 \cdot A_0^{p1-} + A_0^{s0s+} + A_0^{p2s+} - 2 \cdot A_1^{d2+} - 2 \cdot A_1^{p1-}}{32} (s^2 \uparrow u \uparrow u \downarrow + u \downarrow u \uparrow s^2 \uparrow) + \\
& + \frac{A_1^{s0s+} + A_1^{p2s+} + A_0^{s0s+} + A_0^{p2s+}}{16} (u^1 \downarrow u \uparrow s^1 \uparrow + s^1 \uparrow u \uparrow u^1 \downarrow) + \\
& \left. + \frac{A_1^{s0s+} + A_1^{p2s+} - A_0^{s0s+} - A_0^{p2s+}}{16} (u^1 \uparrow u \uparrow s^1 \downarrow + s^1 \downarrow u \uparrow u^1 \uparrow) \right) , \tag{26}
\end{aligned}$$

$$k_{23} \left(\frac{A_1^{s0s+} + A_1^{p2s+}}{4} (u^2 \downarrow u \uparrow s \uparrow + u^2 \downarrow s \uparrow u \uparrow) \right) . \tag{27}$$

Then all members of wave function can be considered. After the grouping of these members we can obtain:

$$u^2 \downarrow u \uparrow s \uparrow \left\{ k_{12} \frac{A_1^{s0+} + A_1^{p2+} + 3A_0^{d2+} + 3A_0^{p1-}}{8} + k_{13} \frac{A_1^{s0s+} + A_1^{p2s+} + 3A_0^{d2s+} + 3A_0^{p1s-}}{8} + k_{23} \frac{A_1^{sos+} + A_1^{p2s+}}{2} \right\} . \tag{28}$$

The left side of the diagram (Fig.2) corresponds to the quark interactions. The right side of Fig.2 determines the zero approximation (first diagram) and the subsequent pair interactions (second diagram). The contribution to $u^2 \downarrow u \uparrow s \uparrow$ is shown in the Fig.3. If we group the same members, we obtain the system integral equations for the Σ state with the $J^p = \frac{5}{2}^+$ of the $(10, 2)$ $(70, 2^+)$ multiplet:

$$\begin{aligned}
A_1^{s0+}(s, s_{12}) &= \lambda b_{1s+}(s_{12}) L_{1s+}(s_{12}) + K_{1s+}(s_{12}) \left[\frac{1}{8} A_1^{s0s+}(s, s_{13}) + \frac{1}{8} A_1^{p2s+}(s, s_{13}) + \frac{3}{8} A_0^{d2s+}(s, s_{13}) \right. \\
&\quad \left. + \frac{3}{8} A_0^{p1s-}(s, s_{13}) + \frac{1}{8} A_1^{s0s+}(s, s_{23}) + \frac{1}{8} A_1^{p2s+}(s, s_{23}) + \frac{3}{8} A_0^{d2s+}(s, s_{23}) + \frac{3}{8} A_0^{p1s-}(s, s_{23}) \right] \\
A_1^{p2+}(s, s_{12}) &= \lambda b_{3d+}(s_{12}) L_{3d+}(s_{12}) + K_{3d+}(s_{12}) \left[\frac{1}{8} A_1^{s0s+}(s, s_{13}) + \frac{1}{8} A_1^{p2s+}(s, s_{13}) + \frac{3}{8} A_0^{d2s+}(s, s_{13}) \right. \\
&\quad \left. + \frac{3}{8} A_0^{p1s+}(s, s_{13}) + \frac{1}{8} A_1^{s0s+}(s, s_{23}) + \frac{1}{8} A_1^{p2s+}(s, s_{23}) + \frac{3}{8} A_0^{d2s+}(s, s_{23}) + \frac{3}{8} A_0^{p1s-}(s, s_{23}) \right] \\
A_1^{s0s+}(s, s_{12}) &= \lambda b_{1s+}(s_{12}) L_{1s+}(s_{12}) + K_{1s+}(s_{12}) \left[\frac{1}{4} A_1^{s0+}(s, s_{13}) + \frac{1}{4} A_1^{p2+}(s, s_{13}) - \frac{1}{8} A_1^{s0s+}(s, s_{13}) \right. \\
&\quad \left. - \frac{1}{8} A_1^{p2s+}(s, s_{13}) + \frac{3}{8} A_0^{d2s+}(s, s_{13}) + \frac{3}{8} A_0^{p1s-}(s, s_{13}) + \frac{1}{4} A_1^{s0+}(s, s_{23}) + \frac{1}{4} A_1^{p2+}(s, s_{23}) \right. \\
&\quad \left. - \frac{1}{8} A_1^{s0s+}(s, s_{23}) - \frac{1}{8} A_1^{p2s+}(s, s_{23}) + \frac{3}{8} A_0^{d2s+}(s, s_{23}) + \frac{3}{8} A_0^{p1s-}(s, s_{23}) \right]
\end{aligned}$$

$$\begin{aligned}
A_1^{p2s+}(s, s_{12}) &= \lambda b_{3_s^{d+}}(s_{12}) L_{3_s^{d+}}(s_{12}) + K_{3_s^{d+}}(s_{12}) \left[\frac{1}{4} A_1^{s0+}(s, s_{13}) + \frac{1}{4} A_1^{p2+}(s, s_{13}) - \frac{1}{8} A_1^{s0s+}(s, s_{13}) \right. \\
&\quad - \frac{1}{8} A_1^{p2s+}(s, s_{13}) + \frac{3}{8} A_0^{d2s+}(s, s_{13}) + \frac{3}{8} A_0^{p1s-}(s, s_{13}) + \frac{1}{4} A_1^{s0+}(s, s_{23}) + \frac{1}{4} A_1^{p2+}(s, s_{23}) \\
&\quad \left. - \frac{1}{8} A_1^{s0s+}(s, s_{23}) - \frac{1}{8} A_1^{p2s+}(s, s_{23}) + \frac{3}{8} A_0^{d2s+}(s, s_{23}) + \frac{3}{8} A_0^{p1s-}(s, s_{23}) \right] \\
A_0^{d2s+}(s, s_{12}) &= \lambda b_{2_s^{d+}}(s_{12}) L_{2_s^{d+}}(s_{12}) + K_{2_s^{d+}}(s_{12}) \left[\frac{1}{4} A_1^{s0+}(s, s_{13}) + \frac{1}{4} A_1^{p2+}(s, s_{13}) + \frac{1}{8} A_1^{s0s+}(s, s_{13}) \right. \\
&\quad + \frac{1}{8} A_1^{p2s+}(s, s_{13}) + \frac{1}{8} A_0^{d2s+}(s, s_{13}) + \frac{1}{8} A_0^{p1s-}(s, s_{13}) + \frac{1}{4} A_1^{s0+}(s, s_{23}) + \frac{1}{4} A_1^{p2+}(s, s_{23}) \\
&\quad \left. + \frac{1}{8} A_1^{s0s+}(s, s_{23}) + \frac{1}{8} A_1^{p2s+}(s, s_{23}) + \frac{1}{8} A_0^{d2s+}(s, s_{23}) + \frac{1}{8} A_0^{p1s-}(s, s_{23}) \right] \\
A_0^{p1s-}(s, s_{12}) &= \lambda b_{1_s^{p-}}(s_{12}) L_{1_s^{p-}}(s_{12}) + K_{1_s^{p-}}(s_{12}) \left[\frac{1}{4} A_1^{s0+}(s, s_{13}) + \frac{1}{4} A_1^{p2+}(s, s_{13}) + \frac{1}{8} A_1^{s0s+}(s, s_{13}) \right. \\
&\quad + \frac{1}{8} A_1^{p2s+}(s, s_{13}) + \frac{1}{8} A_0^{d2s+}(s, s_{13}) + \frac{1}{8} A_0^{p1s-}(s, s_{13}) + \frac{1}{4} A_1^{s0+}(s, s_{23}) + \frac{1}{4} A_1^{p2+}(s, s_{23}) \\
&\quad \left. + \frac{1}{8} A_1^{s0s+}(s, s_{23}) + \frac{1}{8} A_1^{p2s+}(s, s_{23}) + \frac{1}{8} A_0^{d2s+}(s, s_{23}) + \frac{1}{8} A_0^{p1s-}(s, s_{23}) \right].
\end{aligned} \tag{29}$$

Here the function $L_J(s_{ik})$ has the form

$$L_J(s_{ik}) = \frac{G_J(s_{ik})}{1 - B_J(s_{ik})}. \tag{30}$$

The integral operator $K_J(s_{ik})$ is:

$$K_J(s_{ik}) = L_J(s_{ik}) \int_{(m_i+m_k)^2}^{\infty} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^1 \frac{dz}{2}, \tag{31}$$

$$b_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\Lambda_J(i,k)} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J^2(s'_{ik})}{s'_{ik} - s_{ik}}. \tag{32}$$

The function $b_J(s_{ik})$ is the truncate function of Chew-Mandelstam. z is the cosine of the angle between the relative momentum of particles i and k in the intermediate state and the momentum of particle j in the final state, taken in the c.m. of the particles i and k . λ is the current constant. By analogy with the $\Sigma \frac{5}{2}^+(10, 2)$ ($70, 2^+$) state we obtain the rescattering amplitudes of the three various quarks for all $(70, 2^+)$ and $(70, 0^+)$ states which satisfy the system of integral equations (Appendix I).

4. The reduced equations for the $(70, 0^+)$ and $(70, 2^+)$ multiplets.

Let us extract two-particle singularities in $A_J(s, s_{ik})$:

$$A_J(s, s_{ik}) = \frac{\alpha_J(s, s_{ik}) b_J(s_{ik}) G_J(s_{ik})}{1 - B_J(s_{ik})}, \tag{33}$$

$\alpha_J(s, s_{ik})$ is the reduced amplitude. Accordingly to this, all integral equations can be rewritten using the reduced amplitudes. For instance, one considers the first equation of system for the $\Sigma J^p = \frac{5}{2}^+$ of the $(10, 2)$ ($70, 2^+$) multiplet:

$$\begin{aligned}
\alpha_1^{s0+}(s, s_{12}) &= \lambda + \frac{1}{b_{1s^+}(s_{12})} \int_{(m_1+m_2)^2}^{\Lambda_{1+}(1,2)} \frac{ds'_{12}}{\pi} \frac{\rho_{1s^+}(s'_{12}) G_{1s^+}(s'_{12})}{s'_{12} - s_{12}} \\
&\quad \times \int_{-1}^1 \frac{dz}{2} \left(\frac{G_{1s^+}(s'_{13}) b_{1s^+}(s'_{13})}{1 - B_{1s^+}(s'_{13})} \frac{1}{4} \alpha_1^{s0s+}(s, s'_{13}) + \frac{G_{3_s^{d+}}(s'_{13}) b_{3_s^{d+}}(s'_{13})}{1 - B_{3_s^{d+}}(s'_{13})} \frac{1}{4} \alpha_1^{p2s+}(s, s'_{13}) \right. \\
&\quad \left. + \frac{G_{2_s^{d+}}(s'_{13}) b_{2_s^{d+}}(s'_{13})}{1 - B_{2_s^{d+}}(s'_{13})} \frac{3}{4} \alpha_0^{d2s+}(s, s'_{13}) + \frac{G_{1_s^{p-}}(s'_{13}) b_{1_s^{p-}}(s'_{13})}{1 - B_{1_s^{p-}}(s'_{13})} \frac{3}{4} \alpha_0^{p1s-}(s, s'_{13}) \right).
\end{aligned} \tag{34}$$

The connection between s'_{12} and s'_{13} is:

$$s'_{13} = m_1^2 + m_3^2 - \frac{(s'_{12} + m_3^2 - s)(s'_{12} + m_1^2 - m_2^2)}{2s'_{12}} \pm \frac{z}{2s'_{12}} \times \sqrt{(s'_{12} - (m_1 + m_2)^2)(s'_{12} - (m_1 - m_2)^2)} \times \sqrt{(s'_{12} - (\sqrt{s} + m_3)^2)(s'_{12} - (\sqrt{s} - m_3)^2)}. \quad (35)$$

The formula for s'_{23} is similar to (35) with z replaced by $-z$. Thus $A_1^{s_0s+}(s, s'_{13}) + A_1^{s_0s+}(s, s'_{23})$ must be replaced by $2A_1^{s_0s+}(s, s'_{13})$. $\Lambda_J(i, k)$ is the cutoff at the large value of s_{ik} , which determines the contribution from small distances.

The construction of the approximate solution of system of equations is based on the extraction of the leading singularities which are close to the region $s_{ik} = (m_i + m_k)^2$ [29]. Amplitudes with different number of rescattering have the following structure of singularities. The main singularities in s_{ik} are from pair rescattering of the particles i and k . First of all there are threshold square root singularities. Pole singularities are also possible which correspond to the bound states. The diagrams in Fig.2 apart from two-particle singularities have their own specific triangle singularities. Such classification allows us to search the approximate solution of by taking into account some definite number of leading singularities and neglecting all the weaker ones.

We consider the approximation, which corresponds to the single interaction of all three particles (two-particle and triangle singularities) and neglecting all the weaker ones.

The functions $\alpha_J(s, s_{ik})$ are the smooth functions of s_{ik} as compared with the singular part of the amplitude, hence it can be expanded in a series in the singular point and only the first term of this series should be employed further. As s_0 it is convenient to take the middle point of physical region of Dalitz-plot in which $z = 0$. In this case we get from (35) $s_{ik} = s_0 = \frac{s+m_1^2+m_2^2+m_3^2}{m_{12}^2+m_{13}^2+m_{23}^2}$, where $m_{ik} = \frac{m_i+m_k}{2}$. We define the functions $\alpha_J(s, s_{ik})$ and $b_J(s_{ik})$ at the point s_0 . Such a choice of point s_0 allows us to replace integral equations (29) by the algebraic equations for the state Σ with $J^P = \frac{5}{2}^+$ of $(10, 2)$ $(70, 2^+)$:

$$\left\{ \begin{array}{l} \alpha_1^{s_0+} = \lambda + \frac{1}{4} \alpha_1^{s_0s+} M_{1s+1s+} + \frac{1}{4} \alpha_1^{p_2s+} M_{1s+3s+} + \frac{3}{4} \alpha_0^{d_2s+} M_{1s+2s+} + \frac{3}{4} \alpha_0^{p_1s-} M_{1s+1s-} \quad 1s^+ \\ \alpha_1^{p_2+} = \lambda + \frac{1}{4} \alpha_1^{s_0s+} M_{3d+1s+} + \frac{1}{4} \alpha_1^{p_2s+} M_{3d+3s+} + \frac{3}{4} \alpha_0^{d_2s+} M_{3d+2s+} + \frac{3}{4} \alpha_0^{p_1s-} M_{3d+1s-} \quad 3d^+ \\ \alpha_1^{s_0s+} = \lambda + \frac{1}{2} \alpha_1^{s_0+} M_{1s+1s+} + \frac{1}{2} \alpha_1^{p_2+} M_{1s+3d+} - \frac{1}{4} \alpha_1^{s_0s+} M_{1s+1s+} - \frac{1}{4} \alpha_1^{p_2s+} M_{1s+3s+} \quad 1s^+ \\ \quad + \frac{3}{4} \alpha_0^{d_2s+} M_{1s+2s+} + \frac{3}{4} \alpha_0^{p_1s-} M_{1s+1s-} \\ \alpha_1^{p_2s+} = \lambda + \frac{1}{2} \alpha_1^{s_0+} M_{3s+1s+} + \frac{1}{2} \alpha_1^{p_2+} M_{3s+3d+} - \frac{1}{4} \alpha_1^{s_0s+} M_{3s+1s+} - \frac{1}{4} \alpha_1^{p_2s+} M_{3s+3s+} \quad 3s^+ \\ \quad + \frac{3}{4} \alpha_0^{d_2s+} M_{3s+2s+} + \frac{3}{4} \alpha_0^{p_1s-} M_{3s+1s-} \\ \alpha_0^{d_2s+} = \lambda + \frac{1}{2} \alpha_1^{s_0+} M_{2s+1s+} + \frac{1}{2} \alpha_1^{p_2+} M_{2s+3d+} + \frac{1}{4} \alpha_1^{s_0s+} M_{2s+1s+} + \frac{1}{4} \alpha_1^{p_2s+} M_{2s+3s+} \quad 2s^+ \\ \quad + \frac{1}{4} \alpha_0^{d_2s+} M_{2s+2s+} + \frac{1}{4} \alpha_0^{p_1s-} M_{2s+1s-} \\ \alpha_0^{p_1s-} = \lambda + \frac{1}{2} \alpha_1^{s_0+} M_{1s-1s+} + \frac{1}{2} \alpha_1^{p_2+} M_{1s-3d+} + \frac{1}{4} \alpha_1^{s_0s+} M_{1s-1s+} + \frac{1}{4} \alpha_1^{p_2s+} M_{1s-3s+} \quad 1s^- \\ \quad + \frac{1}{4} \alpha_0^{d_2s+} M_{1s-2s+} + \frac{1}{4} \alpha_0^{p_1s-} M_{1s-1s-} , \end{array} \right. \quad (36)$$

Here the reduced amplitudes for the diquarks $1s^+$, $3d^+$, $1s^+$, $3s^+$, $2s^+$, $1s^-$ are given.

We used the following form:

$$M_{X_m^{ip} Y_n^{jq}} \equiv M_{X_m^{ip} Y_n^{jq}}(s, s_0) = I_{X_m^{ip} Y_n^{jq}}(s, s_0) \frac{b_{Y_n^{jq}}(s_0)}{b_{X_m^{ip}}(s_0)}, \quad (37)$$

here X_m^{ip} corresponds to the diquark with total moment X ($X = 0, 1, 2, 3$); $i = s, p, d$ for the s -, p -, d -wave consequently; $p = +, -$ is the p -parity of diquark; $m = s$ for the strange diquark and this index is absent in other case.

The reduced amplitude $\alpha_s^{clmp} \equiv \alpha_s^{clmp}(s, s_0)$ for the $p = +, -$ of parity of diquark; $c = s$ if the diquark is determined as $1s1s$, $c = p$ if we consider $1s1p$ or $1p1p$ states, $c = d$ if we have $1s1d$; $s = 1, 0$ corresponds to the diquark spin ($\uparrow\uparrow, \uparrow\downarrow$), $l = 2, 1, 0, -1$ are the values of projection orbital moment at definite axes, $m = s$ for the strange diquark.

The function $I_{J_1 J_2}(s, s_0)$ takes into account the singularity which corresponds to the simultaneous vanishing of all propagators in the triangle diagrams.

$$I_{J_1 J_2}(s, s_0) = \int_{(m_i + m_k)^2}^{\Lambda_{J_1}} \frac{ds'_{ik}}{\pi} \frac{\rho_{J_1}(s'_{ik}) G_{J_1}^2(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^1 \frac{dz}{2} \frac{1}{1 - B_{J_2}(s_{ij})}. \quad (38)$$

The $G_J(s_{ik})$ functions have the smooth dependence from energy s_{ik} [27] therefore we suggest them as constants. The parameters of model: g_J vertex constants, λ_J cutoff parameters are chosen dimensionless.

$$g_J = \frac{m_i + m_k}{2\pi} G_J, \quad \lambda_J = \frac{4\Lambda_J}{(m_i + m_k)^2}. \quad (39)$$

Here m_i and m_k are quark masses in the intermediate state of the quark loop. Dimensionless parameters g_J and λ_J are supposed to be the constants independent of the quark interaction type. We calculate the system of equations and can determine the mass values of the Σ $J^p = \frac{5}{2}^+$ (10, 2) (70, 2⁺). We calculate a pole in s which corresponds to the bound state of the three quarks.

By analogy with the Σ -hyperon we obtain the system of equations for the reduced amplitudes for all particles (70, 0⁺) and (70, 2⁺) multiplets (Appendix II).

The solutions of the system of equations are considered as:

$$\alpha_J = \frac{F_J(s, \lambda_J)}{D(s)}, \quad (40)$$

where the zeros of the $D(s)$ determinates define of masses of bound states of baryons. $F_J(s, \lambda_J)$ are the functions of s and λ_J . The functions $F_J(s, \lambda_J)$ determine the contributions of subamplitudes to the excited baryon amplitude.

Table I.

The Δ -isobar masses of multiplet (70, 2⁺).

Multiplet	Baryon	Mass (GeV)	Mass (GeV) (exp.)
$\frac{5}{2}^+$ (10, 2)	F_{35}	2.000	2.000
$\frac{3}{2}^+$ (10, 2)	P_{33}	2.088	—

The parameters of model (Tables I-XII): gluon coupling constants $g_s^+ = g_p^- = 0.739$, $g_d^+ = 0.550$, cutoff energy parameters $\lambda = 10.0$, $\lambda_{ss} = 8.9$.

Table II.

The nucleon masses of multiplet (70, 2⁺).

Multiplet	Baryon	Mass (GeV)	Mass (GeV) (exp.)
$\frac{5}{2}^+$ (8, 2)	F_{15}	2.000	2.000
$\frac{3}{2}^+$ (8, 2)	P_{13}	2.045	—
$\frac{7}{2}^+$ (8, 4)	F_{17}	2.074	1.990
$\frac{5}{2}^+$ (8, 4)	F_{15}	2.009	—
$\frac{3}{2}^+$ (8, 4)	P_{13}	1.971	1.900
$\frac{1}{2}^+$ (8, 4)	P_{11}	1.682	—

Table III.

The Σ -hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{5}{2}^+ (10, 2)$	F_{35}	2.049	—
$\frac{3}{2}^+ (10, 2)$	P_{33}	2.179	—
$\frac{5}{2}^+ (8, 2)$	F_{15}	2.057	—
$\frac{3}{2}^+ (8, 2)$	P_{13}	2.118	2.080
$\frac{7}{2}^+ (8, 4)$	F_{17}	2.159	2.030
$\frac{5}{2}^+ (8, 4)$	F_{15}	2.070	2.070
$\frac{3}{2}^+ (8, 4)$	P_{13}	2.033	—
$\frac{1}{2}^+ (8, 4)$	P_{11}	1.660	1.660

Table IV.

The Ξ -hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{5}{2}^+ (10, 2)$	F_{35}	2.135	—
$\frac{3}{2}^+ (10, 2)$	P_{33}	2.280	—
$\frac{5}{2}^+ (8, 2)$	F_{15}	2.143	—
$\frac{3}{2}^+ (8, 2)$	P_{13}	2.212	—
$\frac{7}{2}^+ (8, 4)$	F_{17}	2.258	—
$\frac{5}{2}^+ (8, 4)$	F_{15}	2.154	—
$\frac{3}{2}^+ (8, 4)$	P_{13}	2.109	—
$\frac{1}{2}^+ (8, 4)$	P_{11}	1.659	—

Table V.

The Λ -hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{5}{2}^+ (8, 2)$	F_{15}	2.123	2.110
$\frac{3}{2}^+ (8, 2)$	P_{13}	2.061	—
$\frac{7}{2}^+ (8, 4)$	F_{17}	2.158	—
$\frac{5}{2}^+ (8, 4)$	F_{15}	2.073	—
$\frac{3}{2}^+ (8, 4)$	P_{13}	2.022	—
$\frac{1}{2}^+ (8, 4)$	P_{11}	1.649	—
$\frac{5}{2}^+ (1, 2)$	F_{05}	2.074	—
$\frac{3}{2}^+ (1, 2)$	P_{03}	2.056	—

Table VI.

The Ω -hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{5}{2}^+ (10, 2)$	F_{35}	2.250	—
$\frac{3}{2}^+ (10, 2)$	P_{33}	2.406	—

Table VII.

The Δ -isobar masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{1}{2}^+$ (10, 2)	P_{31}	1.750	1.750

Table VIII.

The nucleon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{1}{2}^+$ (8, 2)	P_{11}	1.710	1.710
$\frac{3}{2}^+$ (8, 4)	P_{13}	1.791	—

Table IX.

The Σ -hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{1}{2}^+$ (10, 2)	P_{31}	1.740	—
$\frac{1}{2}^+$ (8, 2)	P_{11}	1.677	1.770
$\frac{3}{2}^+$ (8, 4)	P_{13}	1.799	—

Table X.

The Ξ -hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{1}{2}^+$ (10, 2)	P_{31}	1.774	—
$\frac{1}{2}^+$ (8, 2)	P_{11}	1.701	—
$\frac{3}{2}^+$ (8, 4)	P_{13}	1.840	—

Table XI.

The Λ -hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{1}{2}^+$ (8, 2)	P_{11}	1.721	1.810
$\frac{3}{2}^+$ (8, 4)	P_{13}	1.800	1.890
$\frac{1}{2}^+$ (1, 2)	P_{01}	1.627	1.600

Table XII.

The Ω -hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (<i>GeV</i>)	Mass (<i>GeV</i>) (exp.)
$\frac{1}{2}^+$ (10, 2)	P_{31}	1.865	—

Table XIII.

Coefficient of Ghew-Mandelstam function for the different diquarks.

	α_J	β_J	δ_J
3^+	$\frac{5}{14}$	$\frac{2}{14} - \frac{5}{14} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$	$-\frac{2}{14}(m_i - m_k)^2$
2^+	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$	0
1^{d+}	$\frac{1}{7}$	$\frac{3}{14} \left(1 + \frac{8m_i m_k}{3(m_i + m_k)^2}\right)$	$-\frac{5}{14}(m_i - m_k)^2$
1^{s+}	$\frac{1}{3}$	$\frac{4m_i m_k}{3(m_i + m_k)^2} - \frac{1}{6}$	$-\frac{1}{6}(m_i - m_k)^2$
0^+	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$	0
0^-	0	$\frac{1}{2}$	$-\frac{1}{2}(m_i - m_k)^2$
1^-	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$	0
2^-	$\frac{3}{10}$	$\frac{1}{5} \left(1 - \frac{3}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}\right)$	$-\frac{1}{5}(m_i - m_k)^2$

5. Calculation results.

The quark masses ($m_u = m_d = m$ and m_s) are not fixed. In order to fix m and m_s , in any way we assume $m = \frac{1}{3}m_\Delta(1.232)$ and $m = \frac{1}{3}m_\Omega(1.672)$ i.e. the quark masses are $m = 0.410 \text{ GeV}$ and $m_s = 0.557 \text{ GeV}$.

The S -wave baryon mass spectra are obtained in good agreement with the experimental data. When we research the excited states the confinement potential can not be neglected. In our case the confinement potential is imitated by the simple increasing of constituent quark masses [30]. The shift of quark mass (parameter $\Delta = 340 \text{ MeV}$) effectively takes into account the changing of the confinement potential. We have shown that inclusion of only gluon exchange does not lead to the appearance of bound states corresponding to the excited baryons in the $(70, 0^+)$ and $(70, 2^+)$ multiplets. The mass shift Δ allows to obtain the mass spectra of these states. The similar result for the P -wave baryons was obtained [24].

In the case considered the same parameters Δ for the u, d, s quarks are chosen. Then the quark masses $m_u = m_d = 0.750 \text{ GeV}$ and $m_s = 0.897 \text{ GeV}$ are given.

In our model the four parameters are used: gluon coupling constants $g_s^+ \equiv g_p^-$ for the s - and p -wave diquarks, g_d^+ for d -wave diquarks, cutoff parameters λ , λ_{ss} for the nonstrange and strange diquarks. Parameter of λ_s was calculated using λ and λ_{ss} parameters.

The parameters $g_s^+ \equiv g_p^- = 0.739$, $g_d^+ = 0.550$, $\lambda = 10.0$, $\lambda_{ss} = 8.9$ have been determined by the baryon masses: $M_{\Delta \frac{1}{2}^+(10,2)} = 1.750 \text{ GeV}$, $M_{N \frac{1}{2}^+(8,2)} = 1.710 \text{ GeV}$, $M_{\Delta \frac{5}{2}^+(10,2)} = 2.000 \text{ GeV}$, and $M_{\Sigma \frac{5}{2}^+(8,4)} = 2.070 \text{ GeV}$.

In the tables I-XII we represent the masses of the nonstrange and strange resonances belonging to the $(70, 0^+)$ and $(70, 2^+)$ multiplets obtained using the fit of the experimental values [25].

The $(70, 0^+)$ and $(70, 2^+)$ multiplets include 414 particles, only 47 baryons have different masses. The 15 resonances are in good agreement with experimental data [25]. We have predicted 32 masses of baryons.

In the framework of the proposed approximate method of solving the relativistic three-particle problem, we have obtained a satisfactory spectrum of $N = 2$ level baryons. The important problem is the mixing the states of baryons and the five quark systems (cryptoexotic baryons [31] or hybrid baryons [32]).

6. Conclusion.

In the papers [22, 23] the relativistic generalization of Faddeev equations in the framework of dispersion relation are constructed. We calculated the S -wave baryon masses using the method based on the extraction of leading singularities of the amplitude. The behavior of electromagnetic form factor of the nucleon and hyperon in the region of low and intermediate momentum transfers is determined by [33]. In the framework of the dispersion relation approach the charge radii of S -wave baryon multiplets with $J^P = \frac{1}{2}^+$ are calculated.

In our paper the relativistic description of three particles amplitudes of P -wave baryons are considered.

We take into account the u, d, s -quarks. The mass spectrum of nonstrange and strange states of multiplet $(\mathbf{70}, \mathbf{1}^-)$ are calculated. We use only four parameters for the calculation of 30 baryon masses. We take into account the mass shift of u, d, s quarks which allows us to obtain the P -wave baryon bound states.

In the present paper the relativistic consideration of three particles amplitudes of $(70, 0^+)$ and $(70, 2^+)$ excited baryons are given. We take into account the u, d, s -quarks. We have calculated the 47 masses of resonances $(70, 0^+)$, $(70, 2^+)$ with only four parameters. We take into account the mass shift (similar to [24]) for u, d, s quarks which allows us to obtain the $N = 2$ level excited baryon bound states. We can see that the masses of these upper multiplets are heavier than lower, that coincides with the nonrelativistic models [34 – 37].

The lowest states $\Lambda(1, 2)(70, 0^+)$ mass is equal to $m = 1.627 \text{ GeV}$. The baryon resonances $(70, 2^+)$ multiplet heavier than ones of the $(70, 0^+)$ multiplet that is similar to the results of the papers [15, 16].

Acknowledgments.

The authors would like to thank T. Barnes, S. Capstick, S. Chekanov, Fl. Stancu for useful discussions. The work was carried with the support of the Russian Ministry of Education (grant 2.1.1.68.26).

Appendix I. The wave functions.

We consider, for instance, the wave functions of the upper submultiplets of decuplet $(10, 2)$ $J^P = \frac{5}{2}^+$, octets $(8, 2)$ $J^P = \frac{5}{2}^+$, $(8, 4)$ $J^P = \frac{7}{2}^+$ and singlet $(1, 2)$ $J^P = \frac{5}{2}^+$, which are corresponded to the projection orbital moment $L_z = 2$. For the lower multiplets one must use the corresponding wave functions. $O(3)$ wave functions possess the mixed symmetry and can be written as:

$$\varphi_{MA}^{O(3)} = \frac{1}{2} (020 - 200 - 101 + 011), \quad \varphi_{MS}^{O(3)} = \frac{1}{\sqrt{12}} (020 + 200 - 2 \cdot 002 - 101 - 011 + 2 \cdot 110), \quad (A1)$$

The $SU(2)$ wave functions have the following form:

$$\varphi_{MA}^{SU(2)} = \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \varphi_{MS}^{SU(2)} = \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow), \quad (A2)$$

The $SU(3)_f$ wave functions are different for each particles.

Multiplet $(10, 2)$.

The totally symmetric $SU(6) \times O(3)$ wave function for each decuplet particle is constructed as:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right) = \frac{1}{\sqrt{2}} \varphi_S^{SU(3)} \left(\varphi_{MA}^{SU(2)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(2)} \varphi_{MS}^{O(3)} \right). \quad (A3)$$

For the Σ^+ -hyperon belonging to the decuplet $SU(3)$ the wave function is:

$$\varphi_S^{SU(3)} = \frac{1}{\sqrt{3}} (usu + suu + uus). \quad (A4)$$

The totally symmetric $SU(6) \times O(3)$ function of Σ^+ from multiplet $(10, 2)$ is given:

$$\begin{aligned} \varphi_{\Sigma^+(10,2)} = \frac{\sqrt{3}}{18} & \left(2\{u^2 \downarrow u \uparrow s \uparrow\} + \{s^2 \downarrow u \uparrow u \uparrow\} - \{u^2 \uparrow u \downarrow s \uparrow\} - \{u^2 \uparrow u \uparrow s \downarrow\} - \{s^2 \uparrow u \uparrow u \downarrow\} - \right. \\ & \left. - 2\{u \downarrow u^1 \uparrow s^1 \uparrow\} - \{s \downarrow u^1 \uparrow u^1 \uparrow\} + \{u \uparrow u^1 \downarrow s^1 \uparrow\} + \{u \uparrow u^1 \uparrow s^1 \downarrow\} + \{s \uparrow u^1 \uparrow u^1 \downarrow\} \right). \end{aligned} \quad (A5)$$

For the Δ^{++} of multiplet $(10, 2)$ the $SU(6) \times O(3)$ wave function can be obtained by the replacement $u \leftrightarrow s$:

$$\varphi_{\Delta^{++}(10,2)} = \frac{1}{6} \left(\{u^2 \downarrow u \uparrow u \uparrow\} - \{u^2 \uparrow u \uparrow u \downarrow\} - \{u \downarrow u^1 \uparrow u^1 \uparrow\} + \{u \uparrow u^1 \uparrow u^1 \downarrow\} \right). \quad (A6)$$

For the Ξ of decuplet $(10, 2)$ the results are similar to Σ by the replacement $u \leftrightarrow s$ or $d \leftrightarrow s$. The Ω^- wave function of $(10, 2)$ coincides with Δ by the replacement $u \rightarrow s$.

Multiplet $(8, 2)$.

The wave functions of octet $(8, 2)$ can be constructed to the following method:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right), \quad (A7)$$

where

$$\varphi_{MA}^{SU(6)} = \frac{1}{\sqrt{2}} \left(\varphi_{MS}^{SU(3)} \varphi_{MA}^{SU(2)} + \varphi_{MA}^{SU(3)} \varphi_{MS}^{SU(2)} \right), \quad (A8)$$

$$\varphi_{MS}^{SU(6)} = \frac{1}{\sqrt{2}} \left(-\varphi_{MS}^{SU(3)} \varphi_{MS}^{SU(2)} + \varphi_{MA}^{SU(3)} \varphi_{MA}^{SU(2)} \right). \quad (A9)$$

In the case of Σ^+ octet the wave functions $\varphi_{MS}^{SU(3)}$ and $\varphi_{MA}^{SU(3)}$ are:

$$\varphi_{MS}^{SU(3)} = \frac{1}{\sqrt{6}} (usu + suu - 2uus), \quad \varphi_{MA}^{SU(3)} = \frac{1}{\sqrt{2}} (usu - suu). \quad (A10)$$

Then the symmetric wave function for Σ^+ of $(8, 2)$ have the following form:

$$\varphi_{\Sigma^+(8,2)} = \frac{\sqrt{3}}{18} (2\{u^2 \uparrow u \downarrow s \uparrow\} + \{s^2 \downarrow u \uparrow u \uparrow\} - \{u^2 \uparrow u \uparrow s \downarrow\} - \{u^2 \downarrow u \uparrow s \uparrow\} - \{s^2 \uparrow u \uparrow u \downarrow\} -$$

$$-2\{u \uparrow u^1 \downarrow s^1 \uparrow\} - \{s \downarrow u^1 \uparrow u^1 \uparrow\} + \{u \uparrow u^1 \uparrow s^1 \downarrow\} + \{u \downarrow u^1 \uparrow s^1 \uparrow\} + \{s \uparrow u^1 \uparrow u^1 \downarrow\}). \quad (A11)$$

The nucleon functions of $(8, 2)$ can be constructed from Σ^+ by the replacement $s \leftrightarrow d$, and the functions of Ξ^0 by the replacement $u \leftrightarrow s$.

In the case of the Λ^0 the $SU(3)$ wave functions $\varphi_{MS}^{SU(3)}$ and $\varphi_{MA}^{SU(3)}$ are:

$$\varphi_{MS}^{SU(3)} = \frac{1}{2} (dsu - usd + sdu - sud), \quad (A12)$$

$$\varphi_{MA}^{SU(3)} = \frac{\sqrt{3}}{6} (sdu - sud + usd - dsu - 2dus + 2uds). \quad (A13)$$

As result, we have obtain the symmetric $SU(6) \times O(3)$ wave function for Λ^0 of $(8, 2)$:

$$\begin{aligned} \varphi_{\Lambda^0(8,2)} = \frac{\sqrt{2}}{12} (&\{u^2 \uparrow d \uparrow s \downarrow\} - \{u^2 \downarrow d \uparrow s \uparrow\} - \{d^2 \uparrow u \uparrow s \downarrow\} + \{d^2 \downarrow u \uparrow s \uparrow\} - \\ & - \{s^2 \uparrow u \uparrow d \downarrow\} + \{s^2 \uparrow u \downarrow d \uparrow\} - \{u \uparrow d^1 \uparrow s^1 \downarrow\} - \{u \downarrow d^1 \uparrow s^1 \uparrow\} + \\ & + \{d \uparrow u^1 \uparrow s^1 \downarrow\} - \{d \downarrow u^1 \uparrow s^1 \uparrow\} + \{s \uparrow u^1 \uparrow d^1 \downarrow\} - \{s \uparrow u^1 \downarrow d^1 \uparrow\}). \end{aligned} \quad (A14)$$

Multiplet $(8, 4)$.

The wave functions of octet $(8, 4)$ are constructed as similar to the cases of $(10, 2)$ and $(8, 2)$ multiplets:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right), \quad (A15)$$

here

$$\varphi_{MA}^{SU(6)} = \varphi_{MA}^{SU(3)} \varphi_S^{SU(2)}, \quad \varphi_{MS}^{SU(6)} = \varphi_{MS}^{SU(3)} \varphi_S^{SU(2)}. \quad (A16)$$

The $SU(2)$ function is totally symmetric:

$$\varphi_S^{SU(2)} = \uparrow\uparrow\uparrow, \quad (A17)$$

and $\varphi_{MS}^{SU(3)}, \varphi_{MA}^{SU(3)}$ similar to $(8, 2)$.

For the Σ^+ of $(8, 4)$ we can calculate:

$$\varphi_{\Sigma^+(8,4)} = \frac{1}{6} (\{s^2 \uparrow u \uparrow u \uparrow\} - \{u^2 \uparrow u \uparrow s \uparrow\} - \{s \uparrow u^1 \uparrow u^1 \uparrow\} + \{u \uparrow u^1 \uparrow s^1 \uparrow\}). \quad (A18)$$

For the nucleon N of $(8, 4)$ the results are similar to Σ^+ of $(8, 4)$ by replacement $s \rightarrow d$; and for Ξ^0 by replacement $u \leftrightarrow s$.

For Λ^0 of $(8, 4)$:

$$\varphi_{\Lambda^0(8,4)} = \frac{\sqrt{6}}{12} (-\{u^2 \uparrow d \uparrow s \uparrow\} + \{d^2 \uparrow u \uparrow s \uparrow\} + \{u \uparrow d^1 \uparrow s^1 \uparrow\} - \{d \uparrow u^1 \uparrow s^1 \uparrow\}). \quad (A19)$$

Multiplet $(1, 2)$.

In the case of Λ_1^0 singlet of $(1, 2)$ the totally symmetric $SU(6) \times O(3)$ function must be constructed in the form:

$$\varphi = \varphi_A^{SU(3)} \varphi_A^{SU(2) \times O(3)}, \quad (A20)$$

$$\varphi_A^{SU(3)} = \frac{1}{\sqrt{6}} (sdu - sud + usd - dsu + dus - uds), \quad (A21)$$

$$\varphi_A^{SU(2) \times O(3)} = \frac{1}{\sqrt{2}} \left(\varphi_{MS}^{SU(2)} \varphi_{MA}^{O(3)} - \varphi_{MA}^{SU(2)} \varphi_{MS}^{O(3)} \right). \quad (A22)$$

Then, we have calculated the $\varphi_{\Lambda_1^0(1,2)}$:

$$\begin{aligned} \varphi_{\Lambda_1^0(1,2)} = & \frac{\sqrt{3}}{6} \left(-\{u^2 \uparrow d \uparrow s \downarrow\} + \{u^2 \uparrow d \downarrow s \uparrow\} + \{d^2 \uparrow u \uparrow s \downarrow\} - \{d^2 \uparrow u \downarrow s \uparrow\} - \right. \\ & -\{s^2 \uparrow u \uparrow d \downarrow\} + \{s^2 \uparrow u \downarrow d \uparrow\} + \{u \uparrow d^1 \uparrow s^1 \downarrow\} - \{u \uparrow d^1 \downarrow s^1 \uparrow\} - \\ & \left. -\{d \uparrow u^1 \uparrow s^1 \downarrow\} + \{d \uparrow u^1 \downarrow s^1 \uparrow\} + \{s \uparrow u^1 \uparrow d^1 \downarrow\} - \{s \uparrow u^1 \downarrow d^1 \uparrow\} \right). \end{aligned} \quad (A23)$$

Appendix II. The system equations of reduced amplitudes of the multiplets $(70, 0^+)$ and $(70, 2^+)$.

Multiplet $(10, 2)$.

$\Delta \frac{5}{2}^+ (10, 2) (70, 2^+)$:

$$\begin{aligned} \alpha_1^{s0+} &= \lambda + \frac{1}{4} \alpha_1^{s0+} M_{1s+1s+} + \frac{1}{4} \alpha_1^{p2+} M_{1s+3d+} + \frac{3}{4} \alpha_0^{d2+} M_{1s+2d+} + \frac{3}{4} \alpha_0^{p1-} M_{1s+1p-} & 1^{s+} \\ \alpha_1^{p2+} &= \lambda + \frac{1}{4} \alpha_1^{s0+} M_{3d+1s+} + \frac{1}{4} \alpha_1^{p2+} M_{3d+3d+} + \frac{3}{4} \alpha_0^{d2+} M_{3d+2d+} + \frac{3}{4} \alpha_0^{p1-} M_{3d+1p-} & 3^{d+} \\ \alpha_0^{d2+} &= \lambda + \frac{3}{4} \alpha_1^{s0+} M_{2d+1s+} + \frac{3}{4} \alpha_1^{p2+} M_{2d+3d+} + \frac{1}{4} \alpha_0^{d2+} M_{2d+2d+} + \frac{1}{4} \alpha_0^{p1-} M_{2d+1p-} & 2^{d+} \\ \alpha_0^{p1-} &= \lambda + \frac{3}{4} \alpha_1^{s0+} M_{1p-1s+} + \frac{3}{4} \alpha_1^{p2+} M_{1p-3d+} + \frac{1}{4} \alpha_0^{d2+} M_{1p-2d+} + \frac{1}{4} \alpha_0^{p1-} M_{1p-1p-} . & 1^{p-} \end{aligned} \quad (A24)$$

$\Sigma \frac{5}{2}^+ (10, 2) (70, 2^+)$:

$$\begin{aligned} \alpha_1^{s0+} &= \lambda + \frac{1}{4} \alpha_1^{s0s+} M_{1s+1s_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{1s+3s_s+} + \frac{3}{4} \alpha_0^{d2s+} M_{1s+2s_s+} + \frac{3}{4} \alpha_0^{p1s-} M_{1s+1p_s-} & 1^{s+} \\ \alpha_1^{p2+} &= \lambda + \frac{1}{4} \alpha_1^{s0s+} M_{3d+1s_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{3d+3s_s+} + \frac{3}{4} \alpha_0^{d2s+} M_{3d+2s_s+} + \frac{3}{4} \alpha_0^{p1s-} M_{3d+1p_s-} & 3^{d+} \\ \alpha_1^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1s_s+1s+} + \frac{1}{2} \alpha_1^{p2+} M_{1s_s+3d+} - \frac{1}{4} \alpha_1^{s0s+} M_{1s_s+1s_s+} - \frac{1}{4} \alpha_1^{p2s+} M_{1s_s+3s_s+} & 1_s^{s+} \\ & + \frac{3}{4} \alpha_0^{d2s+} M_{1s_s+2s_s+} + \frac{3}{4} \alpha_0^{p1s-} M_{1s_s+1p_s-} \\ \alpha_1^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3s_s+1s+} + \frac{1}{2} \alpha_1^{p2+} M_{3s_s+3d+} - \frac{1}{4} \alpha_1^{s0s+} M_{3s_s+1s_s+} - \frac{1}{4} \alpha_1^{p2s+} M_{3s_s+3s_s+} & 3_s^{d+} \\ & + \frac{3}{4} \alpha_0^{d2s+} M_{3s_s+2s_s+} + \frac{3}{4} \alpha_0^{p1s-} M_{3s_s+1p_s-} \\ \alpha_0^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2s_s+1s+} + \frac{1}{2} \alpha_1^{p2+} M_{2s_s+3d+} + \frac{1}{4} \alpha_1^{s0s+} M_{2s_s+1s_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{2s_s+3s_s+} & 2_s^{d+} \\ & + \frac{1}{4} \alpha_0^{d2s+} M_{2s_s+2s_s+} + \frac{1}{4} \alpha_0^{p1s-} M_{2s_s+1p_s-} \\ \alpha_0^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1p_s-1s+} + \frac{1}{2} \alpha_1^{p2+} M_{1p_s-3d+} + \frac{1}{4} \alpha_1^{s0s+} M_{1p_s-1s_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{1p_s-3s_s+} & 1_s^{p-} \\ & + \frac{1}{4} \alpha_0^{d2s+} M_{1p_s-2s_s+} + \frac{1}{4} \alpha_0^{p1s-} M_{1p_s-1p_s-} . \end{aligned} \quad (A25)$$

The $\Xi \frac{5}{2}^+ (10, 2) (70, 2^+)$ reduced equations are similar to the $\Sigma \frac{5}{2}^+ (10, 2) (70, 2^+)$ with the replacement $u \leftrightarrow s$. The $\Omega \frac{5}{2}^+ (10, 2) (70, 2^+)$ reduced equations are constructed by the replacement $s \leftrightarrow u$ for the $\Delta \frac{5}{2}^+ (10, 2) (70, 2^+)$.

The analogous results are obtained if we have considered the spin $J^p = \frac{3}{2}^+ ((70, 2^+))$, $J^p = \frac{1}{2}^+ ((70, 0^+))$.

Multiplet (8, 2).

$N^{\frac{5}{2}+} (8, 2) (70, 2^+)$:

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda - \frac{1}{8} \alpha_1^{s0+} M_{1s+1s+} - \frac{1}{8} \alpha_1^{p2+} M_{1s+3d+} + \frac{3}{8} \alpha_0^{s0+} M_{1s+0s+} + \frac{3}{8} \alpha_0^{p2+} M_{1s+2d+} & 1^{s+} \\
&\quad + \frac{3}{8} \alpha_1^{d2+} M_{1s+3d+} + \frac{3}{8} \alpha_1^{p1-} M_{1s+2p-} + \frac{3}{8} \alpha_0^{d2+} M_{1s+2d+} + \frac{3}{8} \alpha_0^{p1-} M_{1s+1p-} \\
\alpha_1^{p2+} &= \lambda - \frac{1}{8} \alpha_1^{s0+} M_{3d+1s+} - \frac{1}{8} \alpha_1^{p2+} M_{3d+3d+} + \frac{3}{8} \alpha_0^{s0+} M_{3d+0s+} + \frac{3}{8} \alpha_0^{p2+} M_{3d+2d+} & 3^{d+} \\
&\quad + \frac{3}{8} \alpha_1^{d2+} M_{3d+3d+} + \frac{3}{8} \alpha_1^{p1-} M_{3d+2p-} + \frac{3}{8} \alpha_0^{d2+} M_{3d+2d+} + \frac{3}{8} \alpha_0^{p1-} M_{3d+1p-} \\
\alpha_0^{s0+} &= \lambda + \frac{3}{8} \alpha_1^{s0+} M_{0s+1s+} + \frac{3}{8} \alpha_1^{p2+} M_{0s+3d+} - \frac{1}{8} \alpha_0^{s0+} M_{0s+0s+} - \frac{1}{8} \alpha_0^{p2+} M_{0s+2d+} & 0^{s+} \\
&\quad + \frac{3}{8} \alpha_1^{d2+} M_{0s+3d+} + \frac{3}{8} \alpha_1^{p1-} M_{0s+2p-} + \frac{3}{8} \alpha_0^{d2+} M_{0s+2d+} + \frac{3}{8} \alpha_0^{p1-} M_{0s+1p-} \\
\alpha_0^{p2+} &= \lambda + \frac{3}{8} \alpha_1^{s0+} M_{2d+1s+} + \frac{3}{8} \alpha_1^{p2+} M_{2d+3d+} - \frac{1}{8} \alpha_0^{s0+} M_{2d+0s+} - \frac{1}{8} \alpha_0^{p2+} M_{2d+2d+} & 2^{d+} \\
&\quad + \frac{3}{8} \alpha_1^{d2+} M_{2d+3d+} + \frac{3}{8} \alpha_1^{p1-} M_{2d+2p-} + \frac{3}{8} \alpha_0^{d2+} M_{2d+2d+} + \frac{3}{8} \alpha_0^{p1-} M_{2d+1p-} \\
\alpha_1^{d2+} &= \lambda + \frac{3}{8} \alpha_1^{s0+} M_{3d+1s+} + \frac{3}{8} \alpha_1^{p2+} M_{3d+3d+} + \frac{3}{8} \alpha_0^{s0+} M_{3d+0s+} + \frac{3}{8} \alpha_0^{p2+} M_{3d+2d+} & 3^{d+} \\
&\quad - \frac{1}{8} \alpha_1^{d2+} M_{3d+3d+} - \frac{1}{8} \alpha_1^{p1-} M_{3d+2p-} + \frac{3}{8} \alpha_0^{d2+} M_{3d+2d+} + \frac{3}{8} \alpha_0^{p1-} M_{3d+1p-} \\
\alpha_1^{p1-} &= \lambda + \frac{3}{8} \alpha_1^{s0+} M_{2p-1s+} + \frac{3}{8} \alpha_1^{p2+} M_{2p-3d+} + \frac{3}{8} \alpha_0^{s0+} M_{2p-0s+} + \frac{3}{8} \alpha_0^{p2+} M_{2p-2d+} & 2^{p-} \\
&\quad - \frac{1}{8} \alpha_1^{d2+} M_{2p-3d+} - \frac{1}{8} \alpha_1^{p1-} M_{2p-2p-} + \frac{3}{8} \alpha_0^{d2+} M_{2p-2d+} + \frac{3}{8} \alpha_0^{p1-} M_{2p-1p-} \\
\alpha_0^{d2+} &= \lambda + \frac{3}{8} \alpha_1^{s0+} M_{2d+1s+} + \frac{3}{8} \alpha_1^{p2+} M_{2d+3d+} + \frac{3}{8} \alpha_0^{s0+} M_{2d+0s+} + \frac{3}{8} \alpha_0^{p2+} M_{2d+2d+} & 2^{d+} \\
&\quad + \frac{3}{8} \alpha_1^{d2+} M_{2d+3d+} + \frac{3}{8} \alpha_1^{p1-} M_{2d+2p-} - \frac{1}{8} \alpha_0^{d2+} M_{2d+2d+} - \frac{1}{8} \alpha_0^{p1-} M_{2d+1p-} \\
\alpha_0^{p1-} &= \lambda + \frac{3}{8} \alpha_1^{s0+} M_{1p-1s+} + \frac{3}{8} \alpha_1^{p2+} M_{1p-3d+} + \frac{3}{8} \alpha_0^{s0+} M_{1p-0s+} + \frac{3}{8} \alpha_0^{p2+} M_{1p-2d+} & 1^{p-} \\
&\quad + \frac{3}{8} \alpha_1^{d2+} M_{1p-3d+} + \frac{3}{8} \alpha_1^{p1-} M_{1p-2p-} - \frac{1}{8} \alpha_0^{d2+} M_{1p-2d+} - \frac{1}{8} \alpha_0^{p1-} M_{1p-1p-} .
\end{aligned} \tag{A26}$$

$\Sigma^{\frac{5}{2}+} (8, 2) (70, 2^+)$:

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda - \frac{1}{8} \alpha_1^{s0s+} M_{1s+1s_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{1s+3s_d+} + \frac{3}{8} \alpha_0^{s0s+} M_{1s+0s_s+} + \frac{3}{8} \alpha_0^{p2s+} M_{1s+2s_d+} & 1^{s+} \\
&\quad + \frac{3}{8} \alpha_1^{d2s+} M_{1s+3s_d+} + \frac{3}{8} \alpha_1^{p1s-} M_{1s+2s_p-} + \frac{3}{8} \alpha_0^{d2s+} M_{1s+2s_d+} + \frac{3}{8} \alpha_0^{p1s-} M_{1s+1s_p-} \\
\alpha_1^{p2+} &= \lambda - \frac{1}{8} \alpha_1^{s0s+} M_{3d+1s_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{3d+3s_d+} + \frac{3}{8} \alpha_0^{s0s+} M_{3d+0s_s+} + \frac{3}{8} \alpha_0^{p2s+} M_{3d+2s_d+} & 3^{d+} \\
&\quad + \frac{3}{8} \alpha_1^{d2s+} M_{3d+3s_d+} + \frac{3}{8} \alpha_1^{p1s-} M_{3d+2s_p-} + \frac{3}{8} \alpha_0^{d2s+} M_{3d+2s_d+} + \frac{3}{8} \alpha_0^{p1s-} M_{3d+1s_p-} \\
\alpha_1^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1s_s+1s+} + \frac{1}{2} \alpha_1^{p2+} M_{1s_s+3d+} - \frac{5}{8} \alpha_1^{s0s+} M_{1s_s+1s_s+} - \frac{5}{8} \alpha_1^{p2s+} M_{1s_s+3s_d+} & 1s^+ \\
&\quad + \frac{3}{8} \alpha_0^{s0s+} M_{1s_s+0s_s+} + \frac{3}{8} \alpha_0^{p2s+} M_{1s_s+2s_d+} + \frac{3}{8} \alpha_1^{d2s+} M_{1s_s+3s_d+} + \frac{3}{8} \alpha_1^{p1s-} M_{1s_s+2s_p-} \\
&\quad + \frac{3}{8} \alpha_0^{d2s+} M_{1s_s+2s_d+} + \frac{3}{8} \alpha_0^{p1s-} M_{1s_s+1s_p-}
\end{aligned}$$

$$\begin{aligned}
\alpha_1^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{3_s^{d+}3^{d+}} - \frac{5}{8} \alpha_1^{s0s+} M_{3_s^{d+}1_s+} - \frac{5}{8} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} & 3_s^{d+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{3_s^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{3_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \\
&+ \frac{3}{8} \alpha_0^{d2s+} M_{3_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_0^{p1s-} M_{3_s^{d+}1_s^{p-}} \\
\alpha_0^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{0_s^{s+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{0_s^{s+}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{0_s^{s+}1_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{0_s^{s+}3_s^{d+}} & 0_s^{s+} \\
&- \frac{1}{8} \alpha_0^{s0s+} M_{0_s^{s+}0_s^{s+}} - \frac{1}{8} \alpha_0^{p2s+} M_{0_s^{s+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{0_s^{s+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{0_s^{s+}2_s^{p-}} \\
&+ \frac{3}{8} \alpha_0^{d2s+} M_{0_s^{s+}2_s^{d+}} + \frac{3}{8} \alpha_0^{p1s-} M_{0_s^{s+}1_s^{p-}} \\
\alpha_0^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{d+}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{2_s^{d+}1_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{2_s^{d+}3_s^{d+}} & 2_s^{d+} \\
&- \frac{1}{8} \alpha_0^{s0s+} M_{2_s^{d+}0_s^{s+}} - \frac{1}{8} \alpha_0^{p2s+} M_{2_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{2_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{2_s^{d+}2_s^{p-}} \\
&+ \frac{3}{8} \alpha_0^{d2s+} M_{2_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_0^{p1s-} M_{2_s^{d+}1_s^{p-}} \\
\alpha_1^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{3_s^{d+}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{3_s^{d+}1_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} & 3_s^{d+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{3_s^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{3_s^{d+}2_s^{d+}} - \frac{1}{8} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} - \frac{1}{8} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \\
&+ \frac{3}{8} \alpha_0^{d2s+} M_{3_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_0^{p1s-} M_{3_s^{d+}1_s^{p-}} \\
\alpha_1^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{p-}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{p-}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{2_s^{p-}1_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{2_s^{p-}3_s^{d+}} & 2_s^{p-} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{2_s^{p-}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{2_s^{p-}2_s^{d+}} - \frac{1}{8} \alpha_1^{d2s+} M_{2_s^{p-}3_s^{d+}} - \frac{1}{8} \alpha_1^{p1s-} M_{2_s^{p-}2_s^{p-}} \\
&+ \frac{3}{8} \alpha_0^{d2s+} M_{2_s^{p-}2_s^{d+}} + \frac{3}{8} \alpha_0^{p1s-} M_{2_s^{p-}1_s^{p-}} \\
\alpha_0^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{d+}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{2_s^{d+}1_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{2_s^{d+}3_s^{d+}} & 2_s^{d+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{2_s^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{2_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{2_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{2_s^{d+}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{2_s^{d+}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{2_s^{d+}1_s^{p-}} \\
\alpha_0^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1_s^{p-}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{1_s^{p-}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{1_s^{p-}1_s+} - \frac{1}{8} \alpha_1^{p2s+} M_{1_s^{p-}3_s^{d+}} & 1_s^{p-} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{1_s^{p-}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{1_s^{p-}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{1_s^{p-}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{1_s^{p-}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{1_s^{p-}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{1_s^{p-}1_s^{p-}} .
\end{aligned}$$

(A27)

$$\Lambda^{\frac{5}{2}+}(8,2)(70,2^+):$$

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \frac{3}{8} \alpha_1^{s0s+} M_{1^s+1^s+} + \frac{3}{8} \alpha_1^{p2s+} M_{1^s+3_s^{d+}} + \frac{3}{8} \alpha_0^{s0s+} M_{1^s+0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{1^s+2_s^{d+}} & 1_s^{s+} \\
&+ \frac{3}{8} \alpha_1^{d2s+} M_{1^s+3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{1^s+2_s^{p-}} - \frac{1}{8} \alpha_0^{d2s+} M_{1^s+2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{1^s+1_s^{p-}} \\
\alpha_1^{p2+} &= \lambda + \frac{3}{8} \alpha_1^{s0s+} M_{3^{d+}1^s+} + \frac{3}{8} \alpha_1^{p2s+} M_{3^{d+}3_s^{d+}} + \frac{3}{8} \alpha_0^{s0s+} M_{3^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{3^{d+}2_s^{d+}} & 3_s^{d+} \\
&+ \frac{3}{8} \alpha_1^{d2s+} M_{3^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{3^{d+}2_s^{p-}} - \frac{1}{8} \alpha_0^{d2s+} M_{3^{d+}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{3^{d+}1_s^{p-}} \\
\alpha_1^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1^s+1^s+} + \frac{1}{2} \alpha_1^{p2+} M_{1^s+3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{1^s+1^s+} - \frac{1}{8} \alpha_1^{p2s+} M_{1^s+3_s^{d+}} & 1_s^{s+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{1^s+0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{1^s+2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{1^s+3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{1^s+2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{1^s+2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{1^s+1_s^{p-}} \\
\alpha_1^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3_s^{d+}1^s+} + \frac{1}{2} \alpha_1^{p2+} M_{3_s^{d+}3^{d+}} - \frac{1}{8} \alpha_1^{s0s+} M_{3_s^{d+}1^s+} - \frac{1}{8} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} & 3_s^{d+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{3_s^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{3_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{3_s^{d+}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{3_s^{d+}1_s^{p-}} \\
\alpha_0^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{0_s^{s+}1^s+} + \frac{1}{2} \alpha_1^{p2+} M_{0_s^{s+}3^{d+}} + \frac{3}{8} \alpha_1^{s0s+} M_{0_s^{s+}1^s+} + \frac{3}{8} \alpha_1^{p2s+} M_{0_s^{s+}3_s^{d+}} & 0_s^{s+} \\
&- \frac{1}{8} \alpha_0^{s0s+} M_{0_s^{s+}0_s^{s+}} - \frac{1}{8} \alpha_0^{p2s+} M_{0_s^{s+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{0_s^{s+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{0_s^{s+}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{0_s^{s+}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{0_s^{s+}1_s^{p-}} \\
\alpha_0^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{d+}1^s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{d+}3^{d+}} + \frac{3}{8} \alpha_1^{s0s+} M_{2_s^{d+}1^s+} + \frac{3}{8} \alpha_1^{p2s+} M_{2_s^{d+}3_s^{d+}} & 2_s^{d+} \\
&- \frac{1}{8} \alpha_0^{s0s+} M_{2_s^{d+}0_s^{s+}} - \frac{1}{8} \alpha_0^{p2s+} M_{2_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{2_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{2_s^{d+}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{2_s^{d+}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{2_s^{d+}1_s^{p-}} \\
\alpha_1^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3_s^{d+}1^s+} + \frac{1}{2} \alpha_1^{p2+} M_{3_s^{d+}3^{d+}} + \frac{3}{8} \alpha_1^{s0s+} M_{3_s^{d+}1^s+} + \frac{3}{8} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} & 3_s^{d+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{3_s^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{3_s^{d+}2_s^{d+}} - \frac{1}{8} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} - \frac{1}{8} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{3_s^{d+}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{3_s^{d+}1_s^{p-}} \\
\alpha_1^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{p-}1^s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{p-}3^{d+}} + \frac{3}{8} \alpha_1^{s0s+} M_{2_s^{p-}1^s+} + \frac{3}{8} \alpha_1^{p2s+} M_{2_s^{p-}3_s^{d+}} & 2_s^{p-} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{2_s^{p-}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{2_s^{p-}2_s^{d+}} - \frac{1}{8} \alpha_1^{d2s+} M_{2_s^{p-}3_s^{d+}} - \frac{1}{8} \alpha_1^{p1s-} M_{2_s^{p-}2_s^{p-}} \\
&- \frac{1}{8} \alpha_0^{d2s+} M_{2_s^{p-}2_s^{d+}} - \frac{1}{8} \alpha_0^{p1s-} M_{2_s^{p-}1_s^{p-}}
\end{aligned}$$

$$\begin{aligned}
\alpha_0^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{s0s+} M_{2_s^{d+}1_s+} + \frac{3}{8} \alpha_1^{p2s+} M_{2_s^{d+}3_s^{d+}} & 2_s^{d+} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{2_s^{d+}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{2_s^{d+}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{2_s^{d+}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{2_s^{d+}2_s^{p-}} \\
&- \frac{5}{8} \alpha_0^{d2s+} M_{2_s^{d+}2_s^{d+}} - \frac{5}{8} \alpha_0^{p1s-} M_{2_s^{d+}1_s^{p-}} \\
\alpha_0^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1_s^{p-}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{1_s^{p-}3_s^{d+}} + \frac{3}{8} \alpha_1^{s0s+} M_{1_s^{p-}1_s+} + \frac{3}{8} \alpha_1^{p2s+} M_{1_s^{p-}3_s^{d+}} & 1_s^{p-} \\
&+ \frac{3}{8} \alpha_0^{s0s+} M_{1_s^{p-}0_s^{s+}} + \frac{3}{8} \alpha_0^{p2s+} M_{1_s^{p-}2_s^{d+}} + \frac{3}{8} \alpha_1^{d2s+} M_{1_s^{p-}3_s^{d+}} + \frac{3}{8} \alpha_1^{p1s-} M_{1_s^{p-}2_s^{p-}} \\
&- \frac{5}{8} \alpha_0^{d2s+} M_{1_s^{p-}2_s^{d+}} - \frac{5}{8} \alpha_0^{p1s-} M_{1_s^{p-}1_s^{p-}} .
\end{aligned} \tag{A28}$$

Multiplet (8, 4).

$N^{\frac{7}{2}+} (8, 4) (70, 2^+)$:

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \frac{1}{4} \alpha_1^{s0+} M_{1_s+1_s+} + \frac{1}{4} \alpha_1^{p2+} M_{1_s+3_s^{d+}} + \frac{3}{4} \alpha_1^{d2+} M_{1_s+3_s^{d+}} + \frac{3}{4} \alpha_1^{p1-} M_{1_s+2_s^{p-}} & 1_s^{s+} \\
\alpha_1^{p2+} &= \lambda + \frac{1}{4} \alpha_1^{s0+} M_{3_s^{d+}1_s+} + \frac{1}{4} \alpha_1^{p2+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{4} \alpha_1^{d2+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{4} \alpha_1^{p1-} M_{3_s^{d+}2_s^{p-}} & 3_s^{d+} \\
\alpha_1^{d2+} &= \lambda + \frac{3}{4} \alpha_1^{s0+} M_{3_s^{d+}1_s+} + \frac{3}{4} \alpha_1^{p2+} M_{3_s^{d+}3_s^{d+}} + \frac{1}{4} \alpha_1^{d2+} M_{3_s^{d+}3_s^{d+}} + \frac{1}{4} \alpha_1^{p1-} M_{3_s^{d+}2_s^{p-}} & 3_s^{d+} \\
\alpha_1^{p1-} &= \lambda + \frac{3}{4} \alpha_1^{s0+} M_{2_s^{p-}1_s+} + \frac{3}{4} \alpha_1^{p2+} M_{2_s^{p-}3_s^{d+}} + \frac{1}{4} \alpha_1^{d2+} M_{2_s^{p-}3_s^{d+}} + \frac{1}{4} \alpha_1^{p1-} M_{2_s^{p-}2_s^{p-}} . & 2_s^{p-}
\end{aligned} \tag{A29}$$

$\Sigma^{\frac{7}{2}+} (8, 4) (70, 2^+)$:

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \frac{1}{4} \alpha_1^{s0s+} M_{1_s^{s+}1_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{1_s^{s+}3_s^{d+}} + \frac{3}{4} \alpha_1^{d2s+} M_{1_s^{s+}3_s^{d+}} + \frac{3}{4} \alpha_1^{p1s-} M_{1_s^{s+}2_s^{p-}} & 1_s^{s+} \\
\alpha_1^{p2+} &= \lambda + \frac{1}{4} \alpha_1^{s0s+} M_{3_s^{d+}1_s^{s+}} + \frac{1}{4} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{4} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{4} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} & 3_s^{d+} \\
\alpha_1^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{1_s^{s+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{1_s^{s+}3_s^{d+}} - \frac{1}{4} \alpha_1^{s0s+} M_{1_s^{s+}1_s+} - \frac{1}{4} \alpha_1^{p2s+} M_{1_s^{s+}3_s^{d+}} & 1_s^{s+} \\
&+ \frac{3}{4} \alpha_1^{d2s+} M_{1_s^{s+}3_s^{d+}} + \frac{3}{4} \alpha_1^{p1s-} M_{1_s^{s+}2_s^{p-}} \\
\alpha_1^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{3_s^{d+}3_s^{d+}} - \frac{1}{4} \alpha_1^{s0s+} M_{3_s^{d+}1_s+} - \frac{1}{4} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} & 3_s^{d+} \\
&+ \frac{3}{4} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \frac{3}{4} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \\
\alpha_1^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{3_s^{d+}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{3_s^{d+}3_s^{d+}} + \frac{1}{4} \alpha_1^{s0s+} M_{3_s^{d+}1_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} & 3_s^{d+} \\
&+ \frac{1}{4} \alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \frac{1}{4} \alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \\
\alpha_1^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{s0+} M_{2_s^{p-}1_s+} + \frac{1}{2} \alpha_1^{p2+} M_{2_s^{p-}3_s^{d+}} + \frac{1}{4} \alpha_1^{s0s+} M_{2_s^{p-}1_s+} + \frac{1}{4} \alpha_1^{p2s+} M_{2_s^{p-}3_s^{d+}} & 2_s^{p-} \\
&+ \frac{1}{4} \alpha_1^{d2s+} M_{2_s^{p-}3_s^{d+}} + \frac{1}{4} \alpha_1^{p1s-} M_{2_s^{p-}2_s^{p-}} .
\end{aligned} \tag{A30}$$

$\Lambda_{\frac{7}{2}}^{+} (8, 4) (70, 2^{+})$:

$$\begin{aligned}
\alpha_1^{d2+} &= \lambda + \frac{1}{4} \alpha_1^{d2s+} M_{3^d+3_s^d+} + \frac{1}{4} \alpha_1^{p1s-} M_{3^d+2_s^p-} + \frac{3}{4} \alpha_1^{s0s+} M_{3^d+1_s^s+} + \frac{3}{4} \alpha_1^{p2s+} M_{3^d+3_s^d+} & 3^d+ \\
\alpha_1^{p1-} &= \lambda + \frac{1}{4} \alpha_1^{d2s+} M_{2_s^p-3^d+} + \frac{1}{4} \alpha_1^{p1s-} M_{2_s^p-2_s^p-} + \frac{3}{4} \alpha_1^{s0s+} M_{2_s^p-1_s^s+} + \frac{3}{4} \alpha_1^{p2s+} M_{2_s^p-3_s^d+} & 2_s^p- \\
\alpha_1^{d2s+} &= \lambda + \frac{1}{2} \alpha_1^{d2+} M_{3_s^d+3^d+} + \frac{1}{2} \alpha_1^{p1-} M_{3_s^d+2_s^p-} + \frac{1}{2} \alpha_1^{s0s+} M_{3_s^d+1_s^s+} + \frac{1}{2} \alpha_1^{p2s+} M_{3_s^d+3_s^d+} & 3_s^d+ \\
\alpha_1^{p1s-} &= \lambda + \frac{1}{2} \alpha_1^{d2+} M_{2_s^p-3^d+} + \frac{1}{2} \alpha_1^{p1-} M_{2_s^p-2_s^p-} + \frac{1}{2} \alpha_1^{s0s+} M_{2_s^p-1_s^s+} + \frac{1}{2} \alpha_1^{p2s+} M_{2_s^p-3_s^d+} & 2_s^p- \\
\alpha_1^{s0s+} &= \lambda + \frac{1}{2} \alpha_1^{d2+} M_{1_s^s+3^d+} + \frac{1}{2} \alpha_1^{p1-} M_{1_s^s+2_s^p-} + \frac{1}{6} \alpha_1^{d2s+} M_{1_s^s+3_s^d+} + \frac{1}{6} \alpha_1^{p1s-} M_{1_s^s+2_s^p-} & 1_s^s+ \\
&+ \frac{1}{3} \alpha_1^{s0s+} M_{1_s^s+1_s^s+} + \frac{1}{3} \alpha_1^{p2s+} M_{1_s^s+3_s^d+} \\
\alpha_1^{p2s+} &= \lambda + \frac{1}{2} \alpha_1^{d2+} M_{3_s^d+3^d+} + \frac{1}{2} \alpha_1^{p1-} M_{3_s^d+2_s^p-} + \frac{1}{6} \alpha_1^{d2s+} M_{3_s^d+3_s^d+} + \frac{1}{6} \alpha_1^{p1s-} M_{3_s^d+2_s^p-} & 3_s^d+ \\
&+ \frac{1}{3} \alpha_1^{s0s+} M_{3_s^d+1_s^s+} + \frac{1}{3} \alpha_1^{p2s+} M_{3_s^d+3_s^d+} .
\end{aligned} \tag{A31}$$

Multiplet (1, 2).

$\Lambda_{\frac{5}{2}}^{+} (1, 2) (70, 2^{+})$:

$$\begin{aligned}
\alpha_0^{s0+} &= \lambda + \frac{1}{4} \alpha_0^{s0s+} M_{0_s^s+0_s^s+} + \frac{1}{4} \alpha_0^{p2s+} M_{0_s^s+2_s^d+} + \frac{3}{4} \alpha_1^{d2s+} M_{0_s^s+3_s^d+} + \frac{3}{4} \alpha_1^{p1s-} M_{0_s^s+2_s^p-} & 0_s^s+ \\
\alpha_0^{p2+} &= \lambda + \frac{1}{4} \alpha_0^{s0s+} M_{2^d+0_s^s+} + \frac{1}{4} \alpha_0^{p2s+} M_{2^d+2_s^d+} + \frac{3}{4} \alpha_1^{d2s+} M_{2^d+3_s^d+} + \frac{3}{4} \alpha_1^{p1s-} M_{2^d+2_s^p-} & 2^d+ \\
\alpha_0^{s0s+} &= \lambda + \frac{1}{2} \alpha_0^{s0+} M_{0_s^s+0_s^s+} + \frac{1}{2} \alpha_0^{p2+} M_{0_s^s+2^d+} - \frac{1}{4} \alpha_0^{s0s+} M_{0_s^s+0_s^s+} - \frac{1}{4} \alpha_0^{p2s+} M_{0_s^s+2_s^d+} & 0_s^s+ \\
&+ \frac{3}{4} \alpha_1^{d2s+} M_{0_s^s+3_s^d+} + \frac{3}{4} \alpha_1^{p1s-} M_{0_s^s+2_s^p-} \\
\alpha_0^{p2s+} &= \lambda + \frac{1}{2} \alpha_0^{s0+} M_{2_s^d+0_s^s+} + \frac{1}{2} \alpha_0^{p2+} M_{2_s^d+2^d+} - \frac{1}{4} \alpha_0^{s0s+} M_{2_s^d+0_s^s+} - \frac{1}{4} \alpha_0^{p2s+} M_{2_s^d+2_s^d+} & 2_s^d+ \\
&+ \frac{3}{4} \alpha_1^{d2s+} M_{2_s^d+3_s^d+} + \frac{3}{4} \alpha_1^{p1s-} M_{2_s^d+2_s^p-} \\
\alpha_1^{d2s+} &= \lambda + \frac{1}{2} \alpha_0^{s0+} M_{3_s^d+0_s^s+} + \frac{1}{2} \alpha_0^{p2+} M_{3_s^d+2^d+} + \frac{1}{4} \alpha_0^{s0s+} M_{3_s^d+0_s^s+} + \frac{1}{4} \alpha_0^{p2s+} M_{3_s^d+2_s^d+} & 3_s^d+ \\
&+ \frac{1}{4} \alpha_1^{d2s+} M_{3_s^d+3_s^d+} + \frac{1}{4} \alpha_1^{p1s-} M_{3_s^d+2_s^p-} \\
\alpha_1^{p1s-} &= \lambda + \frac{1}{2} \alpha_0^{s0+} M_{2_s^p-0_s^s+} + \frac{1}{2} \alpha_0^{p2+} M_{2_s^p-2^d+} + \frac{1}{4} \alpha_0^{s0s+} M_{2_s^p-0_s^s+} + \frac{1}{4} \alpha_0^{p2s+} M_{2_s^p-2_s^d+} & 2_s^p- \\
&+ \frac{1}{4} \alpha_1^{d2s+} M_{2_s^p-3_s^d+} + \frac{1}{4} \alpha_1^{p1s-} M_{2_s^p-2_s^p-} .
\end{aligned} \tag{A32}$$

References.

1. G.'t Hooft, Nucl. Phys. **B72** 461 (1974).
2. E. Witten, Nucl. Phys. **B160** 57 (1979).
3. J.L. Gervais and B. Sakita, Phys. Rev. Lett. **52** 87 (1984).
4. R. Dashen and A.V. Manohar, Phys. Lett. **B315** 425 (1993).
5. R. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. **D49** 4713 (1994).
6. R. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. **D51** 3697 (1995).
7. C.D. Carone, H. Georgi and S. Osofsky, Phys. Lett. **B322** 227 (1994).
8. M.A. Luty and J. March-Russell, Nucl. Phys. **B426** 71 (1994).
9. E. Jenkins, Phys. Lett. **B315** 441 (1993).
10. E. Jenkins and R.F. Lebed, Phys. Rev. **D52** 282 (1995).
11. J. Dai, R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. **D53** 273 (1996).
12. N. Matagne and Fl. Stancu, Phys. Rev. **D71** 015710 (2005).
13. N. Matagne and Fl. Stancu, arXiv: hep-ph/0610099.
14. N. Matagne and Fl. Stancu, Phys. Lett. **B631** 7 (2005).
15. N. Matagne and Fl. Stancu, Phys. Rev. **D74** 034014 (2006).
16. N. Matagne and Fl. Stancu, arXiv: hep-ph/0603092.
17. I.J.R. Aitchison, J. Phys. **G3** 121 (1977).
18. J.J. Brehm, Ann. Phys. (N.Y.) **108** 454 (1977).
19. I.J.R. Aitchison and J.J. Brehm, Phys. Rev. **D17** 3072 (1978).
20. I.J.R. Aitchison and J.J. Brehm, Phys. Rev. **D20** 1119, 1131 (1979).
21. J.J. Brehm, Phys. Rev. **D21** 718 (1980).
22. S.M. Gerasyuta, Yad. Fiz. **55** 3030 (1992).
23. S.M. Gerasyuta, Z. Phys. **C60** 683 (1993).
24. S.M. Gerasyuta, E.E. Matskevich, Yad. Fiz. **11** (2007), arXiv: hep-ph/0701120.
25. W.M. Yao et al. (Particle Data Group), J. Phys. G **33**, 1 (2006).
26. A.De Rujula, H.Georgi and S.L.Glashow, Phys. Rev. **D12** 147 (1975).
27. V.V. Anisovich, S.M. Gerasyuta and A.V. Sarantsev, Int. J. Mod. Phys. **A6** 625 (1991).
28. G. Chew and S. Mandelstam, Phys. Rev. **119** 467 (1960).
29. V.V. Anisovich and A.A. Anselm, Usp. Fiz. Nauk **88** 287 (1966).
30. S.M. Gerasyuta and I.V. Keltuyala, Yad. Fiz. **54** 793 (1991).
31. S.M. Gerasyuta and I.V. Kochkin, Int. J. Mod. Phys. **E15** 71 (2006).
32. S.M. Gerasyuta and I.V. Kochkin, Phys. Rev. **D66** 116001 (2002).
33. S.M. Gerasyuta and D.V. Ivanov, Vest. SPb University Ser. 4 **11** 3 (1996).
34. N. Isgur and G. Karl, Phys. Rev. **D19** 2653 (1979).
35. A.J. Hey and R.L. Kelly, Phys. Rept. **96** 71 (1983).
36. S. Capstick and N. Isgur, Phys. Rev. **D34** 2809 (1986).
37. S. Capstick and W. Roberts, nucl-th/000828.